Transport Coefficients of Quark Gluon Plasma From Lattice Gauge Theory

S. Sakai[a], A. Nakamura[b] and T.Saito[a]

[a]Faculty of Education, Yamagata University, Yamagata 990 Japan

[b]Research Institute for Information Science and Education, Hiroshima University

Numerical results for the transport coefficients of quark gluon plasma are obtained by lattice simulations on on $16^3 \times 8$ lattice with the quench approximation where we apply the gauge action proposed by Iwasaki. The bulk viscosity is consistent with zero, and the shear viscosity is slightly smaller than the typical hadron masses. They are not far from the simple extrapolation on the figure of perturbative calculation in high temperature limit down to $T \sim T_c$. The gluon propagator in the confined and deconfined phases are also discussed.

1. Introduction

In the phenomenological study of quark gluon plasma(QGP), when its bulk properties are concerned, the system of quarks and gluons is usually treated as gas or liquid. Then the fundamental parameters of QGP such as transport coefficients, are very important information. In the high temperature limit, they are calculated based on the perturbation theory[1–3]. However, as temperature T decreases the perturbative calculation breaks down. The purpose of this work is to calculate the transport coefficient from the fundamental theory of QCD by lattice simulations in the vicinity of T_c.

The calculation of transport coefficients on the lattice is formulated in the framework of linear response theory of Kubo[2,4–6]. They are expressed by the space time integral of retarded Green's function of energy momentum tensors at finite temperature. The shear viscosity η is expressed as,

$$\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\epsilon(t_1-t)} \times \int_{-\infty}^{t_1} dt' < T_{12}(\vec{x},t) T_{12}(\vec{x'},t') >_{ret} \qquad (1)$$

Similarly the bulk viscosity and heat conductivity are expressed in terms of the retarded Green's function of T_{11} and T_{41} components of energy momentum tensor. The direct calculation of the retarded Green's function at finite temperature is very difficult. Then the shortcut is to calculate Matsubara Green's function(G_β) and then by the analytic continuation, we obtain the retarded Green's function at finite temperature. The analytic continuation is carried out by the use of the fact that the spectral function of Fourier transform of the both Green's functions is the same. For the spectral function we use the

following simplest ansatz[6],

$$\rho(\vec{p}=0,\omega) = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2} - \frac{\gamma}{(m+\omega)^2+\gamma^2}\right) \tag{2}$$

where γ is related to the imaginary part of self energy and partially represents the effects of the interactions. Under this ansatz, the transport coefficients are calculated as,

$$\alpha \times a^3 = 2A\frac{2\gamma m}{(\gamma^2+m^2)^2} \tag{3}$$

where α represents η, $\frac{4}{3}\eta+\zeta$ and $\chi \cdot T$, and a is lattice spacing.

Notice that if $m=0$ or $\gamma=0$, transport coefficient becomes zero. In order to determine these three parameters, at least three independent data points in G_β are required in the temperature direction, which means $N_T \geq 6$.

2. Numerical Results on Transport Coefficients of QGP

It is found that the the fluctuation of G_β is large, and it is a very important problem to reduce the fluctuation of G_β. We find that by using the Iwasaki's improved action, the fluctuation is much reduced[8]. Then we apply Iwasaki's Improved action for the simulation of SU(3) gauge theory.

The finite temperature transition point at $N_T=8$ with Iwasaki's improved action is $\beta \sim 2.72$ on $16^3 \times 8$ lattice. We choose our simulation point at $\beta = 3.05, 3.2$ and 3.3. From $0.5 \times 10^6 \sim 10^6$ Monte Carlo data, we obtain $G_\beta(t)$ for T_{11} and T_{12} . But they have still rather large errors. The fit of G_β with parameters by Eq.(2) is done with SALS. The shear and bulk viscosities are obtained by these parameters by Eq.(3), and the errors are estimated by the Jackknife method.

The results for shear and bulk viscosities are shown in Fig.1. It is found that the bulk viscosity is smaller than shear viscosity and is consistent with zero within errors. This is consistent with the result of perturbative calculation at high temperature limit, which has been $\zeta = 0$[1–3].

We compare our results for shear viscosity with those perturbative formula at high temperature limit[1–3], $\eta = C \cdot T^3/(-\alpha_s^2 log\alpha_s)$. Where α_s is a running coupling constants given by, $\alpha_s = \frac{2\pi}{11}log(\frac{T}{\Lambda})$, Λ is a scale parameter of QCD. C is a constant depending on the method of calculation ($0.06 \leq C \leq 0.25$). By using the relation $T = 1/(N_T \times a)$, perturbative formula is expressed by the lattice spacing a as follows,

$$\eta a^3 = C/(-N_T^3\alpha_s^2 log\alpha_s) \tag{4}$$

The scale parameter Λ on the lattice is determined by assuming asymptotic scaling relation and β_c at $N_T=8$. From the finite size scaling relation reported by the Tsukuba group[9], $\beta_c \simeq 2.74$ at $V=\infty$ for $N_T=8$. And using $T_c \simeq 276MeV$ [9] for improved action, Λ is determined by, $\Lambda/T_c \simeq 1.5$. With these quantities the perturbative formula Eq.(4) is compared with our calculation as shown in Fig.2. It is found that our result are located not so different from the simple extrapolation of perturbative calculation on the figure. But notice that the Eq.(4) breaks down around $T/T_c \simeq 1$.

The shear viscosity in the physical unit are obtained if we assume asymptotic scaling

Figure 1. Shear and Bulk viscosity

Figure 2. Shear viscosity from numerical and perturbative calculation

relation for $\beta \geq 2.74$ region. The results are shown in Fig.3. They are slightly smaller than the ordinally hadron masses. What is the physical effects on the phenomenology of quark gluon plasma, when it has shear viscosity with this magnitudes, is a very interesting problem.

In our calculation, G_β of T_{14} from which the heat conductivity is calculated, has large background and we could not get signal from it.

3. Gluon Propagators at Finite Temperature

At first stage of our calculation we planed to calculate transport coefficients both on confined and deconfined phase. From the simulation of U(1) gauge theory, it is found that the fluctuation of G_β in the confined phase is much larger than those of deconfined phase, and we could not determine the Green's functions even with about 1.5×10^6 data[10] in confined phase. Similar results are obtained in the case of SU(2) gauge theory. To find the the difference of G_β between confined and deconfined phase, we study the gluon propagators at finite temperature on $8^3 \times 4$ lattice, because G_β are expressed by the gauge invariant combination of gluon propagators. However the gluon propagator itself is gauge dependent, and we should fix the gauge. It is done by maximize $I = Re(\sum Tr(U_\mu))$ by successive gauge transformation[11], which is a lattice version of Lorentz gauge.

The stopping condition for maximizing I is $R = |I^{n+1} - I^n|/I^n \leq 10^{-14}$. It is found that the gluon propagators are very sensitive to R when $R \geq 10^{-9}$. With this stopping condition, we could not find the gauge copies in the deconfined phase. While in the confined phase, gauge copies are found. Then in the confined phase, we repeat random gauge transformation and gauge fixing processes 5 times, and calculate the gluon propagators on the gauge fixed configuration with largest values of I. In Fig.4, the transverse gluon propagators are shown for confined and deconfined phase which has a momentum of $\vec{p} = (2\pi/8, 0, 0)$ and propagate in the z-direction which is 8 in lattice size.

In the deconfined phase, the gluon propagator is well represented by the the free Lorentz

Figure 3. Shear viscosity in physical unit

Figure 4. Gluon propagator in Confined and deconfined phase

gauge gluon propagator on the lattice. While in the confined phase, the propagator shows quite different behavior as first found by Nakamura[12] at the large distance with standard action. In Fig.4 it is found that the effective mass of gluon seems to increase with the distance. What is a physical meaning of this behavior should further be investigated, but we find that this is a reason why the Matsubara Green's function of energy momentum tensor is so noisy in the confined phase.

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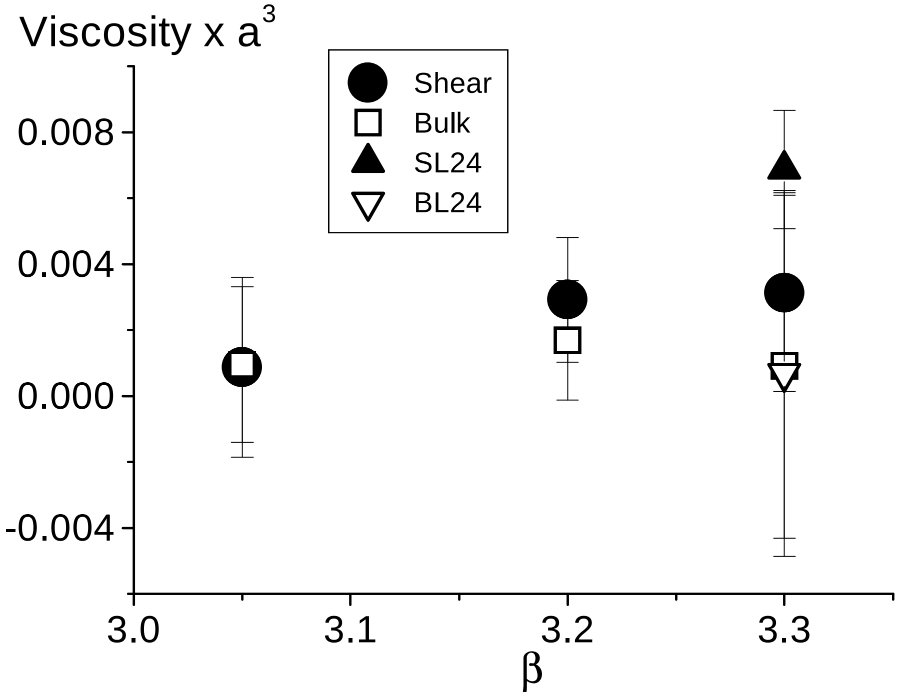

Fig.1 Shear and Bulk Viscosity

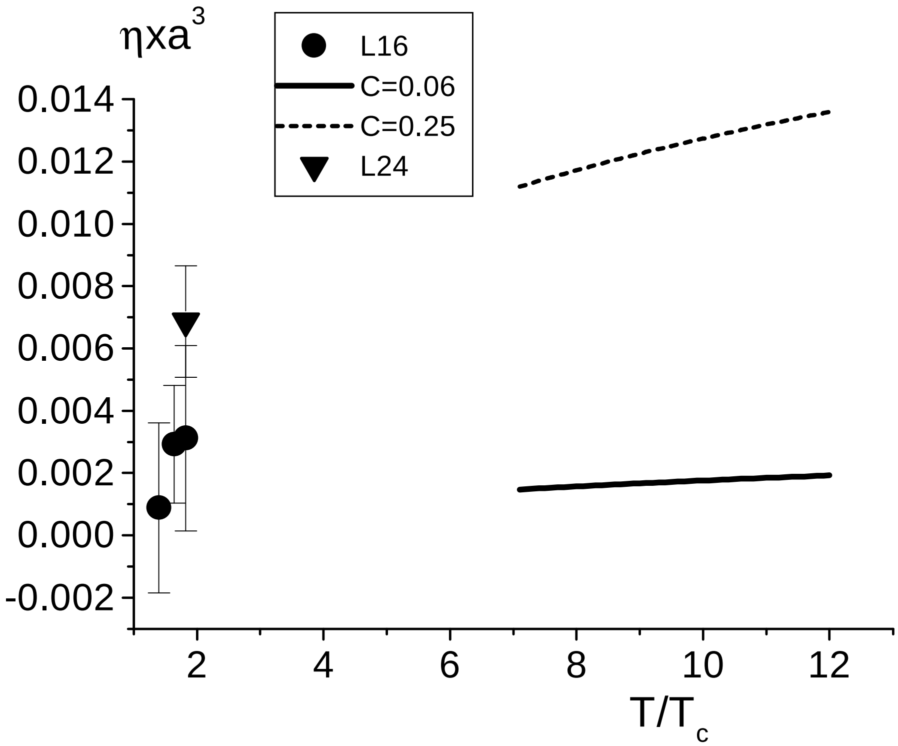

Fig.2 Shear Viscosity from Numerical and Perturbative Calculation

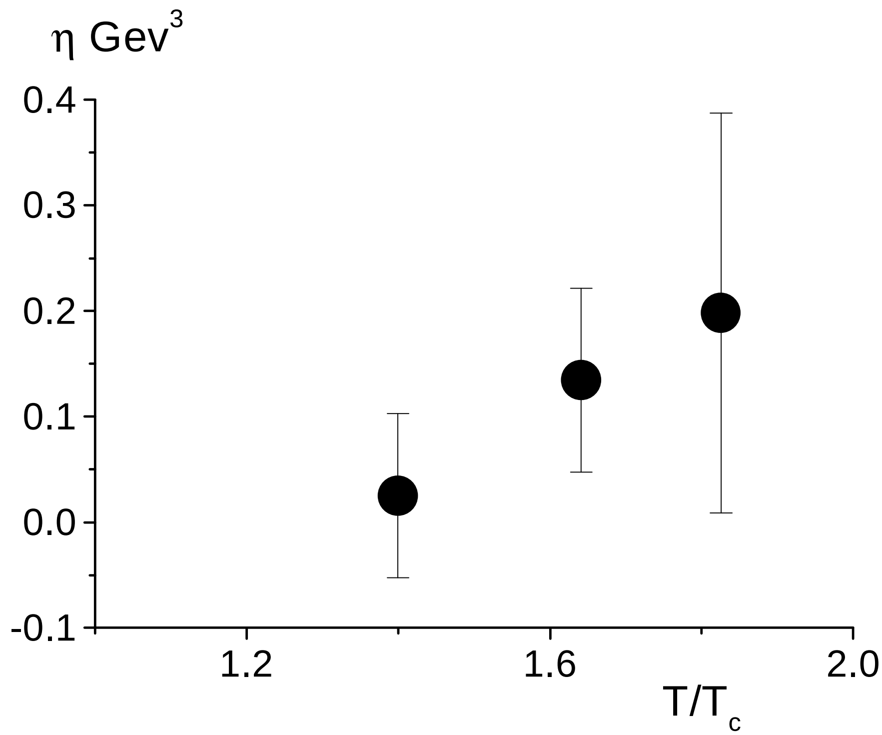

Fig.3 Shear Viscosity in Physical Unit

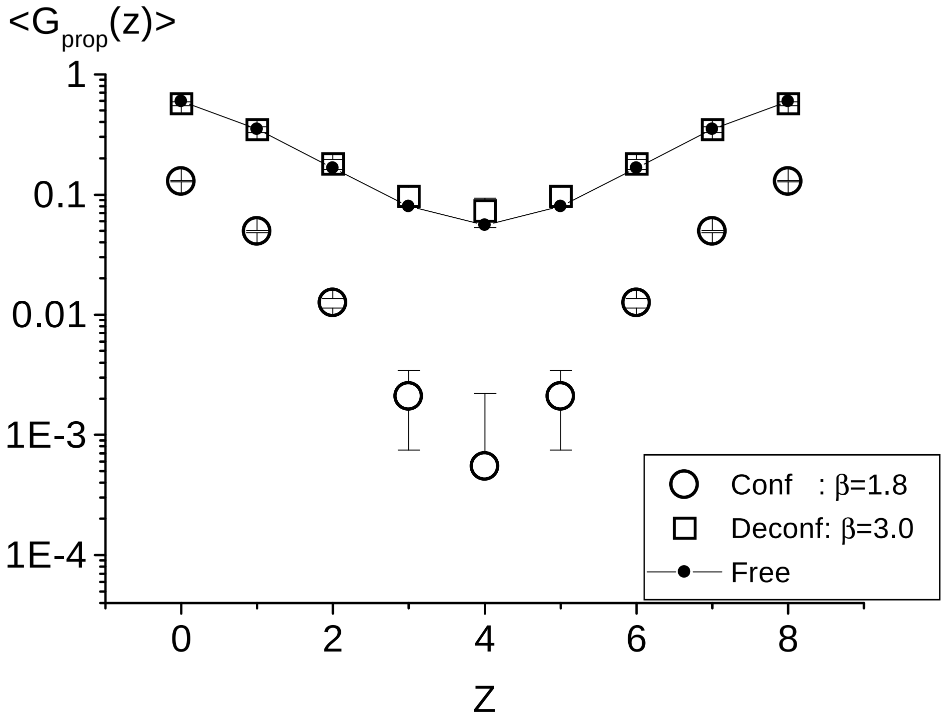

Fig.4 Gluon Propagator in Confined And Deconfined Phase